Journal of Engineering Physics and Thermophysics, Vol. 76, No. 1, 2003

RADIATION PRESSURE ON AN ACOUSTIC CYLINDER IN THE INTERMEDIATE AND FAR ZONES OF AN ULTRASONIC FIELD

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UDC 620.179.16

The field of an acoustic transducer in the region of detection of an ultrasonic wave beyond the near zone is considered as applied to nondestructive check of the surface density of foil-film materials by the method of their irradiation with an ultrasonic wave through air. The results obtained refine the existing concept of the pressure distribution on the surface of an acoustic cylinder in the intermediate and far zones of the field.

One promising physical method of automated check of the surface density of foil-film materials moving in the technological flow is based on their direct irradiation with an ultrasonic wave through air [1, 2]. The field which is traversed by the checked material is produced in air by an acoustic transducer (radiator) (AT) of long-duration ultrasonic pulses. The pressure of the wave transmitted by the material is perceived by the detector, which is coaxial with and similar to the acoustic transducer. The pressure, averaged over the cross section of the acoustic cylinder (whose base is an acoustic transducer of radius a), serves as an informative signal of the checked parameter of the irradiated material. Its pattern is determined with account for the axial $P(0)$ (on the axis of the acoustic transducer) and boundary $P_{\rm a}$ pressures on the surface of the cylinder with diameter $2a$.

An approximate formula of the axial pressure of the acoustic transducer is given in [3–5]. A more exact solution of the wave equation for $P(0)$ [6] has the form

$$P\,(0) = P_0\left[1 - \frac{z + r_{\rm m}}{2r_{\rm m}}\exp\,(-\,i\varphi_P)\right]\exp\,(-\,ikz)\,, \tag{1}$$

where $P_0 = \rho c v_{\rm n}$, $k = 2\pi/\lambda$, $r_{\rm m} = \sqrt{z^2 + a^2}$, and $\varphi_P = k(r_{\rm m} - z)$. According to formula (1), the field on the axis of the acoustic transducer is characterized by the presence of the pressure minima and maxima, the last of which is at a distance of

$$z_{\rm near} = a^2/\lambda \tag{2}$$

from the acoustic transducer.

It is customary to term the region $z = 0$–$z_{\rm near}$ the near-field zone. The wave transmitted by the material is usually detected in air at a distance of one to three times $z_{\rm near}$ from the acoustic transducer.

Until recently, the wave pressure beyond the near-field zone at some point Q, which is at a distance q from the axis of the acoustic transducer, had been determined [4] in terms of the Bessel function of first order J_1 as $P_q = iP_0\dfrac{\pi a^2}{\lambda r_{\rm m}}\dfrac{2J_1(\varepsilon_q)}{\varepsilon_q}$, where the argument $\varepsilon_q = kaq/r_{\rm m}$. On the surface of the acoustic cylinder, $q = a$ and the pressure on the surface is

$$P_{\rm a} = iP_0\,J_1\left(\frac{ka^2}{r_{\rm m}}\right). \tag{3}$$

The proximity of this expression is obvious from the fact that for arguments equal to the roots of the Bessel function $P_{\rm a}$ vanishes, which is consistent with experimental data [5] Consequently, a more accurate (than (3)) determination of the considered pressure is necessary.

GIREDMET, Moscow, Russia Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 76, No. 1, pp. 134–137, January–February, 2003. Original article submitted August 30, 2001; revision submitted June 24, 2002.

1062-0125/03/7601-0158$25.00

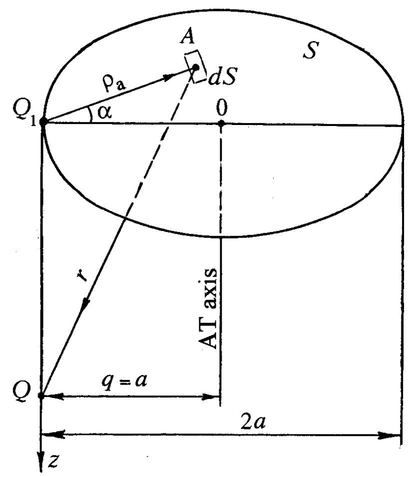

Fig. 1. Schematic of the piston acoustic transducer.

The general solution of the wave equation for pressure P_q at the point Q of the field of the piston acoustic transducer [6] in the function of the potential

$$\psi = \frac{1}{r} \exp(-ikr) \tag{4}$$

of point sources has the form

$$P_q = \frac{P_0}{4\pi} \iint_S \left(ik\psi r - z \frac{\partial \psi}{\partial r} \right) \frac{dS}{r} . \tag{5}$$

We place the origin of the coordinate system (z, ρ_a, α) on the projection Q_1 at the edge of the acoustic transducer (Fig. 1). The distance r between the points of radiation A and detection Q is related to the radius vector ρ_a of the point of radiation by the relation $r^2 = z^2 + \rho_a^2$, the differentiation of which yields $rdr = \rho_a d\rho_a$ and correspondingly $dS = \rho_a d\rho_a d\alpha = rdrd\alpha$. In the selected coordinate system, the integration variable α changes from $-\pi/2$ to $\pi/2$, the radius vector ρ_a of the point A at the center of the radiation element dS changes from 0 to $\rho_e = 2a \cos\alpha$, and correspondingly, the integration variable r changes from z to

$$r_e = \sqrt{z^2 + 4a^2 \cos^2\alpha} . \tag{6}$$

With account for this fact and for the quantity $ikr\psi = -\frac{\partial}{\partial r}(\psi r)$ obtained from differentiation of (4), from the general solution (5) there follows the expression for the pressure on the surface of the acoustic cylinder

$$P_a = -\frac{P_0}{4\pi} \int_{-\pi/2}^{\pi/2} d\alpha \int_z^{r_e} \frac{\partial}{\partial r} [\psi (r+z)]\, dr = \frac{P_0}{4\pi} \int_{-\pi/2}^{\pi/2} \left[2 \exp(-ikz) - \left(1 + \frac{z}{r_e} \right) \exp(-ikr_e) \right] d\alpha$$

or

$$P_a = \frac{P_0}{2} \exp(-ikz) - P_d , \tag{7}$$

where the first term is half the pressure of the plane wave of direct radiation and the second term is the diffraction pressure interfering with direct radiation:

$$P_{\mathrm{d}} = \frac{P_0}{2\pi} \int_0^{\pi/2} \left(1 + \frac{z}{r_{\mathrm{e}}}\right) \exp\left(-ikr_{\mathrm{e}}\right) d\alpha \,. \tag{8}$$

For distances z from the acoustic transducer which exceed the length of the near-field zone, it is expedient to represent r_{e} determined from (6) as

$$r_{\mathrm{e}} = r_0 \sqrt{1 + n \cos\beta}\,, \tag{9}$$

where $\beta = 2\alpha$, $r_0 = \sqrt{z^2 + 2a^2}$, and $n = 2a^2/r_0^2$.

In expanding in a series, in the exponent index of the integrand in (8) we consider three terms of the series

$$kr_{\mathrm{e}} = kr_0 \sqrt{1 + n\cos\beta} = kr_0 \left(1 + \frac{n}{2}\cos\beta - \frac{n^2}{8}\cos^2\beta\right)$$

instead of two.

Upon the substitution $\cos^2\beta = 1 - \sin^2\beta$ we obtain

$$\exp\left(-ikr_{\mathrm{e}}\right) = \left[\exp\left(-ikr_{\mathrm{e}}\right) \exp\left(-i\frac{\varepsilon n}{4}\sin^2\beta\right)\right], \tag{10}$$

where $\varepsilon_0 = kr_0\left(1 - \frac{n^2}{8}\right)$ and $\varepsilon = \frac{n}{2}kr_0 = \frac{ka^2}{r_0}$. In the required expansion of the second exponent in a series it is sufficient to consider the first two terms of the series $\exp\left(-i\frac{\varepsilon n}{4}\sin^2\beta\right) = 1 - \frac{\varepsilon n}{4}\sin^2\beta$ due to the smallness of the values of the integrals with subsequent even powers of $\sin\beta$. Since z/z_{e} in (8) is a "slowly changing" function, we restrict ourselves to consideration of two terms of the expansion series of this parameter:

$$\frac{z}{r_{\mathrm{e}}} = \frac{z}{r_0\sqrt{1 + n\cos\beta}} = \frac{z}{r_0} - \frac{nz}{2r_0}\cos\beta\,.$$

With account for these expansions, the introduction of the integration variable $\beta = 2\alpha$, $d\alpha = \frac{1}{2}d\beta$, and the limits of integration for β going from 0 to π, expression (8) for the diffraction pressure is

$$P_{\mathrm{d}} = \frac{P_0}{4\pi}\exp\left(-i\varepsilon_0\right)\int_0^{\pi} B \exp\left(-i\varepsilon\cos\beta\right) d\beta\,,$$

where

$$B = \left(1 + \frac{z}{r_0} - \frac{nz}{2r_0}\cos\beta\right)\left(1 - i\frac{\varepsilon n}{4}\sin^2\beta\right) = 1 + \frac{z}{r_0} - \frac{n}{2}\left[\frac{z}{r_0}\cos\beta + i\frac{\varepsilon}{2}\left(1 + \frac{z}{r_0}\right)\sin^2\beta\right] + i\frac{n^2 z}{8r_0}\varepsilon\cos\beta\sin^2\beta\,.$$

According to the theory of cylindrical functions [7], we have

$$\int_0^{\pi}\exp\left(-i\varepsilon\cos\beta\right)d\beta = \pi J_0(\varepsilon)\,, \quad \int_0^{\pi}\exp\left(-i\varepsilon\cos\beta\right)d\beta = -i\pi J_1(\beta)\,,$$

$$\varepsilon\int_0^{\pi}\exp\left(-i\varepsilon\cos\beta\right)\sin^2\beta d\beta = \pi J_1(\varepsilon)\,, \quad \varepsilon\int_0^{\pi}\exp\left(-i\varepsilon\cos\beta\right)\cos\beta\sin^2\beta = -i\pi J_2(\varepsilon)\,.$$

TABLE 1. Calculated Dependences of the Pressure Moduli on the Acoustic Cylinder $|P_a/P_0|$ on the Reduced Distance z/z_{near} According to the Refined (13) and Approximate (3) Formulas for the Transducer with ka = 16

z/z_{near}	$100\cdot\lvert P_a/P_0\rvert$ according to:		$100\cdot\frac{\Delta P}{P_0}$	z/z_{near}	$100\cdot\lvert P_a/P_0\rvert$ according to		$100\cdot\frac{\Delta P}{P_0}$
	(13)	$\left\lvert J_1\left(\frac{ka^2}{r_m}\right)\right\rvert$			(13)	$\left\lvert J_1\left(\frac{ka^2}{r_m}\right)\right\rvert$	
1	48	30.3	– 1.73	3.468	56.3	58.1	1.8
1.166	53.3	34.6	– 1.87	4	55.3	56.5	1.2
1.592	33.3	0	– 33.3	5	50.2	51.1	0.9
2	36.8	30.7	– 6.1	7	40	40.4	0.4
2.505	50	50.2	0.2	10	29.7	29.8	0.1
3	55.2	57	1.8				

Consequently, the diffraction pressure is

$$P_d = \frac{P_0}{4}\exp(-i\varepsilon_0)\left[\left(1+\frac{z}{r_0}\right)J_0(\varepsilon) - i\,\frac{n}{4}\left(1-\frac{z}{r_0}\right)J_1(\varepsilon) + \frac{n^2 z}{8r_0}J_2(\varepsilon)\right]. \tag{11}$$

Here the coefficients of $J_1(\varepsilon)$ and $J_2(\varepsilon)$ beyond the near-field zone (when $z \ge z_{near}$) are much less than unity in magnitude. Therefore, the last two components of the diffraction pressure in (11) can be neglected and

$$4P_d = P_0\,\frac{z+r_0}{4r_0}\,J_0(\varepsilon)\exp(-i\varepsilon_0)\,. \tag{12}$$

Thus, according to (7), the pressure on the acoustic cylinder beyond the near-field zone is

$$P_a = \frac{P_0}{2}\left[1 - \frac{z+r_0}{2r_0}\,J_0(\varepsilon)\exp(-i\varphi_P)\right]\exp(-ikz)\,, \tag{13}$$

where the phase is

$$\varphi_P = \varepsilon_0 - kz = k\,(r_0 - z) - \frac{n}{4}\,\varepsilon\,. \tag{14}$$

The dependence of the pressure modulus for the acoustic transducer with the most often used wave parameter ka = 16 on the reduced distance z/z_{near} is given in Table 1 compared to the values calculated from the approximate formula (3).

It is obvious from the data of the table that, in contrast to the axial pressure determined from (1), the pressure on the cylinder still experiences oscillations beyond the near-field zone, thus reaching the last minimum at z = $1.592z_{near}$ and the last maximum at z = $3.468z_{near}$; then, with further increase in z/z_{near}, the pressure decreases monotonously. By virtue of this, the two-zone (near and far zones) gradation of the field can be supplemented with the intermediate zone of the field lying between the last maxima of pressure: on the axis and on the acoustic cylinder of the acoustic transducer (when z ranges from z_{near} to $3.5z_{near}$).

The pressure obtained according to the approximate formula (3) from [4] is sharply understated in the first portion of the intermediate zone and overstated in the second portion of it, thus approaching, as is clear from $\Delta P/P_0$ in the table, the values obtained in the far field according to (13).

The dependence (13) obtained in the present work can be assumed to be a refined solution of the wave equation of pressure on the acoustic cylinder in the intermediate and far zones of the transducer field.

NOTATION

a, radius of the acoustic transducer; c, velocity of propagation of ultrasound; dS, element of the acoustic-transducer surface; dS_e, surface element at the edge of the acoustic transducer; k, wave number; J_0, J_1, and J_2, Bessel functions; P and $P(0)$, ultrasonic pressure and pressure on the axis of the acoustic transducer; P_0 and P_q, pressure of the radiated plane wave and at the point Q; Q_1, projection of the point Q onto the surface of the acoustic transducer; q, distance between the point Q and the axis of the acoustic transducer; r, distance between the points A and Q; r_m, distance between the center of the detector and the edge of the acoustic transducer; r_e, distance between the point Q and the edge of the acoustic transducer; S, radiation surface; v_n, normal component of the vibrational speed of the acoustic transducer; z, coordinate, projection of r onto the axis of the acoustic transducer; z_{near}, length of the near-field zone; α, angular coordinate; β, integration variable; ε, argument of the Bessel function; ε_q, phase argument; λ, wave-length; ρ, density of the medium (air); ρ_a, radius vector of the radiation point; ρ_e, radius vector of the element dS_e at the edge of the acoustic transducer; φ_P, phase of pressure of the detected wave. Subscripts: a, acoustic; near, near-field zone; e, edge; m, maximum; n, normal component; 0, zero value; d, diffraction.

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